Soret effect on mixed convection heat and mass transfer along a semi-infinite horizontal plate in the presence of heat generation and chemical reaction

Mostafa A.A. Mahmoud

Abstract: The aim of this work is to study Soret and chemical reaction effects on laminar mixed convection boundary-layer flow and heat and mass transfer past a semi-infinite horizontal flat plate in the presence of heat generation. A transformation is introduced that reduces the equations governing the flow to a system of nonlinear coupled ordinary differential equations. The transformed equations are solved numerically using the fourth-order Runge–Kutta integration scheme coupled with the shooting method. Graphical results for the velocity, temperature, and concentration profiles for various parametric conditions are presented and studied. The effects of the chemical reaction parameter, Soret parameter, and the heat generation parameter on the local skin-friction coefficient, the local Nusselt number, and the local Sherwood number are presented in tabular form and discussed.

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Résumé : Nous étudions ici les résultats de l'effet Soret et des réactions chimiques sur l'écoulement d'une couche limite laminaire avec mélange convectif et le transfert de chaleur et de masse, en contact avec une surface horizontale plane chauffée. Nous introduisons une transformation qui réduit les équations décrivant le système à un système d'équations différentielles ordinaires non linéaires couplées. Les équations sont solutionnées à l'aide d'une approche Runge–Kutta au quatrième ordre avec méthode de tir. Nous présentons sous forme graphique et analysons nos résultats pour les profils de vitesse, de température et de concentration pour différentes valeurs des paramètres. Nous présentons sous forme de table et analysons les effets des paramètres de réaction chimique, de Soret et de génération de chaleur sur le coefficient de friction de peau, le nombre de Nusselt et le nombre local de Sherwood.

[Traduit par la Rédaction]

1. Introduction

Because of their important applications in natural and industrial manufacturing processes, heat and mass transfer problems combined with chemical reaction have received much attention in recent decades. The effects of a chemical reaction on the boundary-layer flow and heat and mass transfer have been investigated by many authors [1–10]. Combined forced and free convection or mixed convection arises in many transport processes in nature and engineering devices. Atmospheric boundary-layer flow, heat exchangers, solar collector, nuclear reactors, and electronic equipment are examples in which the effect of forced flow on a buoyant flow is significant. The problem of mixed convection of a free stream flowing over a stationary horizontal plate has been studied first by Mori [11] and Sparrow and Minkowycz [12] using a perturbation series in terms of the buoyancy parameter. In particular, self-similar solutions have been studied by Schneider [13], Merkin and Ingham [14], and Mahmoud [15] for a wall temperature prescribed as an inverse square root of the distance from the leading edge. The Soret effect, also called thermo-diffusion or thermal-diffusion, corresponds to species differentiation developing in an initial homogeneous mixture subjected to a thermal gradient. Because of the importance of the Soret effect, particularly in isotope separation and mixtures between gases with very light molecular weight (such as H_2 or He) or medium molecular weight (such as air), many investigators have studied and reported results for these flows, for example, Kafoussias [16], Nanousis [17], and Sattar et al. [18]. In many engineering and physical problems in which fluid undergoes exothermic or endothermic reaction, one needs to consider the temperature-dependent heat generation or absorption that may exert strong influence on the heat transfer characteristics. Following Foraboschi and Federico [19], we shall assume that the volumetric rate of heat generation is

$$Q = \begin{cases} Q_0\,(T - T_\infty)\,, & T \geq T_\infty \\ 0, & T < T_\infty \end{cases} \tag{1}$$

where Q_0 is the heat generation or absorption coefficient. Many authors have studied the effect of heat generation or absorption

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M.A.A. Mahmoud. Department of Mathematics, Faculty of Science, Benha University (13518), Egypt
(e-mail: mostafabdelhameed@yahoo.com).

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on boundary-layer heat transfer. For examples please see, Kelly et al. [20], Nataraja et al. [21], Snoth et al. [22], Abel et al. [23], Chamkha [24, 25], Molla et al. [26], and Mahmoud and Megahed [27]. The aim of this work is to study the Soret effect on mixed convection boundary-layer flow past a horizontal flat plate in the presence of a chemical reaction and heat generation. Applications of this problem can be found in the area of nuclear reactors, solar collectors, power production, the cooling of electronic equipment, and energy-storage devices.

2. Formulation of the problem

Consider a semi-infinite horizontal flat plate aligned parallel with a uniform free stream with velocity u_∞, density ρ_∞, temperature T_∞, and concentration C_∞. The coordinates are chosen such that x represents the distance along the plate from the leading edge and y represents the distance normal to the plate. The plate is maintained at a temperature $T_{\mathrm{w}}(x)$ and concentration $C_{\mathrm{w}}(x)$. The flow over the plate is considered to be planar, laminar, and steady. Here, we assume that the diffusion-thermo (Dufour) effect is negligible. The viscous dissipation term in the energy equation is also neglected. Fluid properties are assumed to be constant except for the density variation that induces the buoyancy forces. With this assumption and the application of the Boussinesq approximations, the continuity, momentum, energy, and mass diffusion equations in the presence of the Soret effect, a first-order chemical reaction, and heat generation, are given by [28]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \tag{3}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} + g\beta\left(T - T_\infty\right) + g\beta^*\left(C - C_\infty\right) = 0 \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p}\left(T - T_\infty\right) \tag{5}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + D_T\frac{\partial^2 T}{\partial y^2} - k_1\left(C - C_\infty\right) \tag{6}$$

where u and v are the velocities in the x and y directions, respectively, ν is the kinematic viscosity, g is the acceleration of gravity, β is the thermal expansion coefficient, β^* is the expansion coefficient for concentration, T is the temperature of the fluid, p is the pressure, K is the thermal conductivity, D is the mass diffusivity, C is the concentration, k_1 is the rate of chemical reaction, D_T is the thermal diffusivity, and c_p is the specific heat at constant pressure.

The appropriate boundary conditions are

$$y = 0: u = 0, \quad v = 0, \quad T = T_{\mathrm{w}}(x), \quad C = C_{\mathrm{w}}(x),$$
$$y \to \infty: u \to u_\infty, \quad T \to T_\infty, \quad C \to C_\infty \tag{7}$$

It is convenient to transform the governing equations into a local similar dimensionless form, which can be suitable for solution as an initial value problem. This can be done by introducing the stream function ψ that satisfies the continuity equation (2) and is defined in the usual manner such that,

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \tag{8}$$

and using

$$\eta = \left(\frac{u_\infty}{\nu x}\right)^{1/2} y, \quad \psi = \left(u_\infty \nu x\right)^{1/2} f(\eta), \quad \theta = \frac{T - T_\infty}{T_{\mathrm{w}} - T_\infty},$$
$$\varphi = \frac{C - C_\infty}{C_{\mathrm{w}} - C_\infty}, \quad T_{\mathrm{w}} = T_\infty + Ax^{1/2},$$
$$C_{\mathrm{w}} = C_\infty + Bx^{1/2} \tag{9}$$

where A and B are constants. Eliminating the pressure gradient term and substituting the transformation given in (9) into (3)–(6), one obtains the following system of nonlinear ordinary differential equations governing the flow and the heat and mass transport,

$$2f''' + ff'' + \lambda\eta\theta + \lambda^*\eta\varphi = 0 \tag{10}$$

$$2\theta'' + Pr\left(f'\theta + f\theta' + \gamma\theta\right) = 0 \tag{11}$$

$$2\varphi'' + Sc\left(f'\varphi + f\varphi'\right) + 2ScSr\theta'' - 2D_a Sc\varphi = 0 \tag{12}$$

where

$\lambda = \left(gA\beta/u_\infty^2\right)\left(\nu/u_\infty\right)^{1/2}$	buoyancy parameter,
$\lambda^* = \left(gB\beta^*/u_\infty^2\right)\left(\nu/u_\infty\right)^{1/2}$	modified buoyancy parameter,
$Pr = \nu/\alpha$	Prandtl number,
$Sc = \nu/D$	Schmidt number,
$D_a = k_1 x/u_\infty$	local chemical parameter,
$\gamma = 2xQ_0/u_\infty \rho c_p$	local heat generation parameter,
$Sr = D_T A/B$	Soret parameter.

To eliminate x from γ and D_a, Q_0 and k_1 must be selected such that they are inversely proportional to the distance along the plate. Therefore, (10)–(12) represent locally similar equations.

The transformed boundary conditions are given by

$$\eta = 0: f = 0, \qquad f' = 0, \quad \theta = 1, \qquad \varphi = 1,$$
$$\eta \to \infty: f' \to 1, \qquad \theta \to 0, \qquad \varphi \to 0 \tag{13}$$

Important physical parameters for this flow and the heat and mass transfer situation are the local skin-friction coefficient C_{f_x}, the local Nusselt number Nu_x, and the local Sherwood number Sh_x.

These are defined by

$$C_{f_x} = \frac{2\tau_{\mathrm{w}}}{\rho_\infty u_\infty^2} = \left(Re_x x\right)^{-(1/2)} f''(0), \tag{14}$$

$$Nu_x = \frac{xq_{\mathrm{w}}}{K\left(T_{\mathrm{w}} - T_\infty\right)} = -{Re_x}^{1/2}\theta'(0) \tag{15}$$

$$Sh_x = \frac{xq_{\mathrm{w}}^*}{D(C_{\mathrm{w}} - C_\infty)} = -{Re_x}^{1/2}\varphi'(0) \tag{16}$$

where

$$\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -K\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$q_w^* = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

and $Re_x = (ux/\nu)$ is the local Reynolds number.

3. Numerical solution

The system of nonlinear ordinary differential equations (10)–(12) with the boundary conditions (13) has been solved numerically using a fourth-order Runge–Kutta integration scheme along with the Newton–Raphson shooting method. Equations (10), (11), and (12), together with the boundary conditions (13), are solved by converting them to an initial value problem. We set,

$$y_1 = f, \qquad y_2 = y_1', \qquad y_3 = y_2', \qquad y_4 = \theta$$
$$y_5 = y_4', \qquad y_6 = \varphi, \quad \text{and} \quad y_7 = y_6'$$

Then, (11), (12), and (13) are reduced to a system of a first-order ordinary differential equations, i.e.,

$$y_1' = y_2, \qquad y_1(0) = 0$$
$$y_2' = y_3, \qquad y_2(0) = 0$$
$$y_3' = -\frac{1}{2}\left[y_1 y_3 + \lambda\eta y_4 + \lambda^*\eta y_6\right], \qquad y_3(0) = \epsilon_1$$
$$y_4' = y_5, \qquad y_4(0) = 1$$
$$y_5' = -\frac{Pr}{2}[y_2 y_4 + y_1 y_5 + \gamma y_4], \qquad y_5(0) = \epsilon_2$$
$$y_6' = y_7, \qquad y_6(0) = 1$$
$$y_7' = -\frac{Sc}{2}(y_2 y_6 + y_1 y_7) - Sr\,Sc\,y_5' + D_a Sc\,y_6, \qquad y_7(0) = \epsilon_3$$

where ϵ_1, ϵ_2, and ϵ_3 are determined such that the outer boundary conditions $y_2(\infty)$, $y_4(\infty)$, and $y_6(\infty)$ are satisfied.

To assess the accuracy of this procedure, a solution is used to obtain the local skin-friction coefficient $f''(0)$ at $\lambda^* = 0$, $\gamma = 0$, and $Pr = 1$. We note that our results are in excellent agreement with those reported by Pop and Gorla (Newtonian case, where $n = 1$) [29] as shown in Table 1.

Table 1. Comparisons of $f''(0)$ for various values of λ with $\lambda^* = 0$, $\gamma = 0$, and $Pr = 1$.

λ	Pop and Gorla [29]	Present study
0	0.3320	0.3320
0.2	0.5525	0.5525
0.5	0.7757	0.7756
1	1.0574	1.0575

4. Results and discussion

The value of the Prandtl number Pr is chosen for air ($Pr = 0.71$). The values of Schmidt number Sc are taken for hydrogen ($Sc = 0.22$), helium ($Sc = 0.3$), water vapor ($Sc = 0.60$), oxygen ($Sc = 0.66$), and ammonia ($Sc = 0.78$). The value of the buoyancy parameter is chosen to be $\lambda = 0.5$ and the value of the modified buoyancy parameter is chosen to be $\lambda^* = 0.1$, since these positive values represent cooling of the plate. The effect of the local heat generation parameter γ and the local chemical parameter D_a on the dimensionless velocity $f'(\eta)$ are shown in Figs. 1 and 2. Figure 1 shows that the overshoot of the velocity increases, accompanied by a location of the velocity peak nearer to the wall as the heat-generation parameter γ increases. This overshoot in the velocity distribution can be physically explained, because the internal heat generation results in an increase in the buoyancy forces, which in turn induce more flow along the plate. On the other hand, it can be seen from Fig. 2 that the reverse is true when increasing the chemical parameter. This is because the chemical reaction parameter has the tendency to reduce the concentration profile of the fluid along the plate. This causes the modified buoyancy force to decrease, resulting in less flow along the plate. Also, from Figs. 1 and 2 it can be seen that the velocity increases as γ increases, and decreases as D_a increases. The dimensionless temperature distribution θ for different values of the parameter γ are presented in Fig. 3. It can be seen from Fig. 3 that the temperature gradient at the wall and the overshoot of the temperature increase as γ increases. From Fig. 4, a higher chemical reaction parameter D_a causes a quicker decrease of the dimensionless concentration across the boundary layer. The shape of concentration profiles in Fig. 5 shows thinner concentration boundary layers are found in response to higher Schmidt numbers Sc. Figure 6 shows that the dimensionless concentration distribution decreases as Sr increases near the wall and increases away from the wall. Taking $\gamma = 0$ and integrating (12) with the boundary conditions (13), one notes that $\theta'(0) = 0$ for all values of λ, λ^*, and Pr. This means that in the absence of heat generation, there is no local heat transfer at the plate surface for all $x > 0$. In this case, all the heat necessary to change the fluid temperature must be transferred to the singular point $x = 0$ [13].

Table 2 illustrates the effects of the Schmidt number, the chemical parameter, and the heat generation parameter on the local skin-friction coefficient, the local Nusselt number, and local Sherwood number. From Table 2, it is observed that the local skin-friction coefficient decreases as the chemical parameter, or the Schmidt number increases, while the local skin-friction coefficient increases as the heat generation parameter or Soret number increases. As for the chemical parameter, the heat generation parameter or the Schmidt number increases the local Nusselt number Nu_x increases, while Nu_x decreases as the Soret number increases. The local Sherwood number increases as the chemical parameter, the heat generation parameter, the Schmidt number, or Soret number increases.

5. Conclusion

The effects of chemical reaction, Soret number, and heat generation on the flow and the heat and mass transfer on a lami-

Fig. 1. Velocity profiles for various values of γ with $D_a = 4$, $Sc = 0.22$, $Pr = 0.71$, and $Sr = 0.1$.

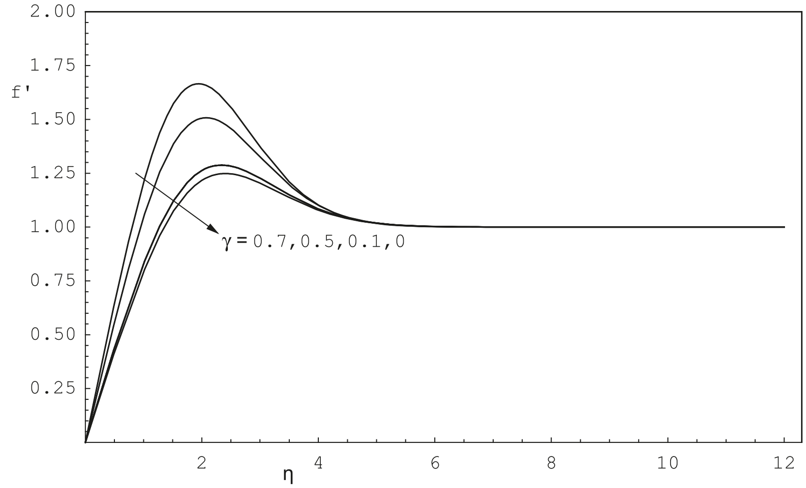

Fig. 2. Velocity profiles for various values of D_a with $\gamma = 0.1$, $Sc = 0.22$, $Pr = 0.71$, and $Sr = 0.1$.

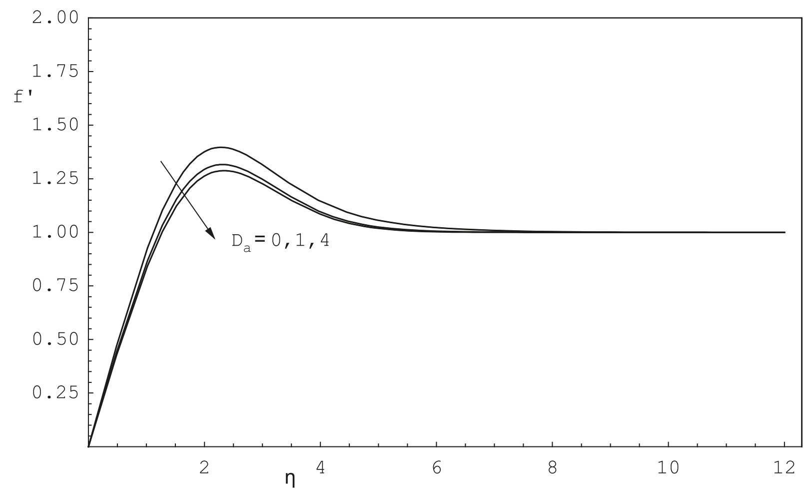

Fig. 3. Temperature profiles for various values of γ with $D_a = 4$, $Pr = 0.71$, $Sc = 0.22$, and $Sr = 0.1$.

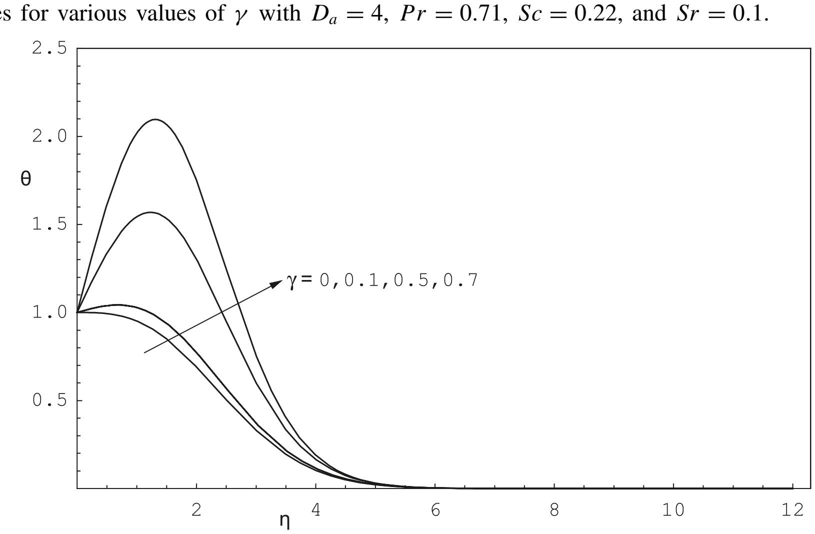

Fig. 4. Concentration profiles for various values of D_a with $\gamma = 0.1$, $Sc = 0.22$, $Pr = 0.71$, and $Sr = 0.1$.

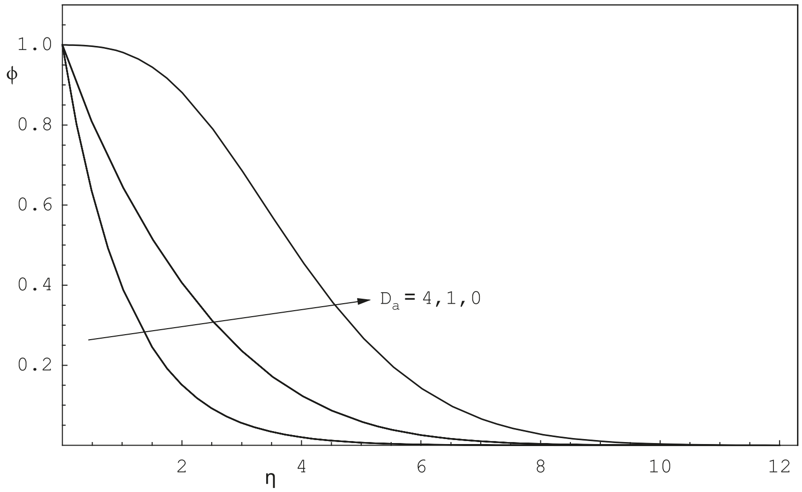

Fig. 5. Concentration profiles for various values of Sc with $D_a = 4$, $Pr = 0.71$, $\gamma = 0.1$, and $Sr = 0.1$.

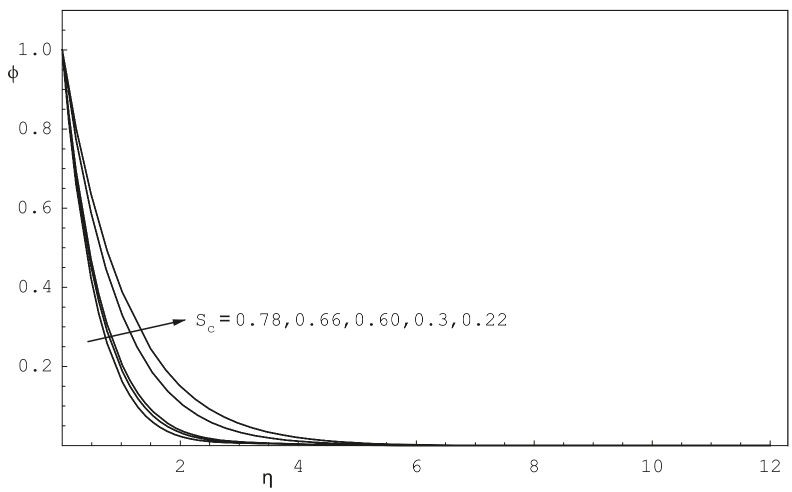

Fig. 6. Concentration profiles for various values of Sr with $\gamma = 0.1$, $Sc = 0.22$, $Pr = 0.71$, and $D_a = 4$.

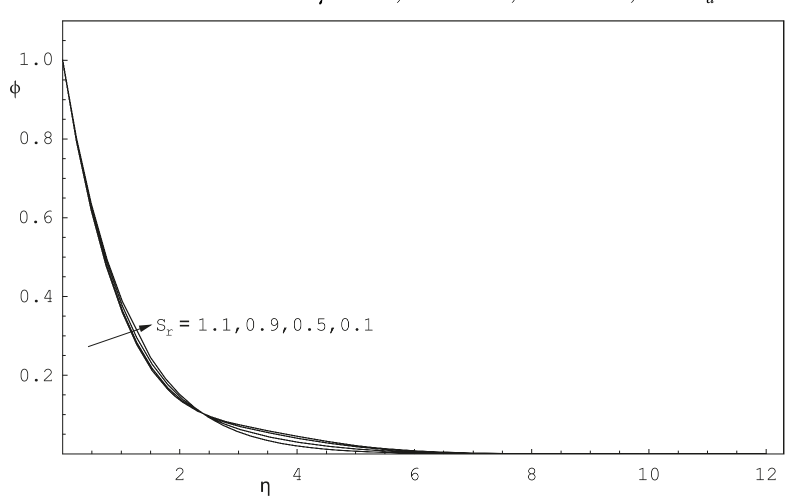

Table 2. Values of $f''(0)$, $-\theta'(0)$, and $-\varphi'(0)$ for various values of the parameters.

Sr	D_a	Sc	γ	$f''(0)$	$-\theta'(0)$	$-\varphi'(0)$
0.1	4	0.22	0.7	1.310 98	−1.333 41	0.939 35
0.1	4	0.22	0.5	1.130 49	−0.747 83	0.934 45
0.1	4	0.22	0.1	0.893 53	−0.100 27	0.929 76
0.1	4	0.22	0.1	0.893 53	−0.100 27	0.929 76
0.1	4	0.30	0.1	0.889 30	−0.100 45	1.087 65
0.1	4	0.60	0.1	0.882 21	−0.100 73	1.543 01
0.1	4	0.66	0.1	0.881 39	−0.100 75	1.618 87
0.1	4	0.78	0.1	0.880 23	−0.100 79	1.760 85
0.1	0	0.22	0.1	0.982 46	−0.096 13	0.002 48
0.1	1	0.22	0.1	0.920 57	−0.099 02	0.437 26
0.1	4	0.22	0.1	0.893 53	−0.100 27	0.929 76
0.1	4	0.22	0.1	0.893 53	−0.100 27	0.929 76
0.5	4	0.22	0.1	0.893 83	−0.100 24	0.945 52
0.9	4	0.22	0.1	0.894 06	−0.100 21	0.961 28
1.1	4	0.22	0.1	0.894 16	−0.100 20	0.969 17

nar mixed convection boundary-layer flow past a horizontal flat plate have been investigated. The governing equations are transformed into a system of nonlinear ordinary differential equation by means of local similarity variables. This system of equations is solved numerically. The effects of the chemical parameter, the Schmidt number, Soret number, and heat generation parameter on the local skin-friction coefficient, the local Nusselt number, and the local Sherwood number are discussed. It was found that the local skin-friction coefficient decreases as the chemical parameter or the Schmidt number increases, while the local skin-friction coefficient increases as the heat generation parameter increases. Also, the local Sherwood number increased as either the chemical parameter or the Schmidt number was increased and decreases as the heat generation parameter increases. The chemical parameter and the Schmidt number have the effect of reducing the local Nusselt number while the heat generation parameter enhanced the local Nusselt number.

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